GRADE : 12

SUBJECT : Mathematics

TITLE : June P I

EXAMINER : Mr A. Slaughter

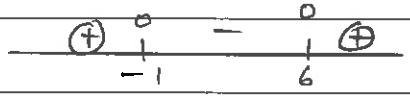
TOTAL MARKS : 150

DATE : ___ / 06 / 20 19

SOLUTIONS

TIME : 3 hour(s)

1.1.1.	$3x^2 = 5x$		
	$3x^2 - 5x = 0$ ✓	std form	
	$x(3x-5) = 0$ ✓	factors	
	$x = 0$ or $x = 5/3$ ✓	both	3
1.1.2.	$2x - \frac{3}{x} = 7$		
	LCD : x $(\because x \neq 0)$		
	x thru		
	$2x^2 - 3 = 7x$		
	$2x^2 - 7x - 3 = 0$ ✓	std form	
	$(\quad x \quad) = 0$ xx		
	$\therefore x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$ ✓	sub into formula	
		• no sub shown max 3/4	
	$= \frac{7 \pm \sqrt{73}}{4}$		
	$= 3,89$ or $-0,39$ ✓ ✓	each x	4

1.1.3	$3x^{-2/5} = 0,81$		
	$x^{-2/5} = 0,27$		
	$(x^{-2/5})^{-5/2} = \pm (0,27)^{-5/2}$ ✓	$-5/2$	
	$x = \pm$ ✓ $26,40$ ✓		3
1.1.4	$x(x-5) > 6$		
	$x^2 - 5x - 6 > 0$ ✓	std form	
	$(x+1)(x-6) > 0$ ✓	factors	
			
	$\therefore$ $x < -1$ or $6 < x$ ✓✓	• $x > -1$ or $x > 6$ $\checkmark_2$ • and/$\cup$ $\checkmark_1$ • $\leq$ $\checkmark_1$	4
1.2.	$1 = 2y - x$		
	$\therefore x = 2y - 1$ ✓		
	$(2y-1)^2 - (2y-1)y + y^2 = 7$ ✓	sub	
	$(2y-1)(2y-1) - (2y^2 - y) + y^2 = 7$		
	$4y^2 - 4y + 1 - 2y^2 + y + y^2 - 7 = 0$		
	$3y^2 - 3y - 6 = 0$		
	$\div 3$: $y^2 - y - 2 = 0$ ✓	std form	
	$(y+1)(y-2) = 0$ ✓	factors	

• $(x+1)(x-2) = 0$
bd max 3/6

	$\therefore y = -1$ or 2 ✓	both y's	
	$\therefore x = 2(-1) - 1$ or $2(2) - 1$		
	$= -3$ $\quad = 3$ ✓	both x's	
	$\therefore x = -3$ and $y = -1$		
	or		
	$x = 3$ and $y = 2$ ⟶		6
1.3.1.	$\frac{3^{2020}}{3^{2014} - 3^{2018}}$		
	$= \frac{3^{2014+6}}{3^{2014} \cdot 3^{2014+4}}$		
	$= \frac{3^{2014} \cdot 3^6}{3^{2014} - 3^{2014}\, 3^4}$		
	$= \frac{3^{2014} \cdot 3^6}{3^{2014}(1 - 34)}$ ✓	com fact	
	$= \frac{729}{1 - 81}$		
	$= -\frac{729}{80}$ ✓ ⟶	answer	2
1.3.2.	$9^{x+1} + 26\,3^x = 3$		
	$(3^2)^{x+1} + 26.3^x = 3$		
	$3^{2x+2} + 26.3^x = 3$		
	$3^{2x} \cdot 3^2 + 26.3^x = 3$		
	$9.3^{2x} + 26.3^x - 3 = 0$ ✓	std form	
	$k = 3^x$		

	$9k^2 + 26k - 3 = 0$		
	$(9k - 1)(k + 3) = 0$ ✓	factors	
	$\therefore k = \frac{1}{9}$ or -3 ✓	both	
	$\therefore 3^x = \frac{1}{3^2}$ $3^x = -3$		
	$3^x = 3^{-2}$ no soln ✓		
	$x = -2$ ✓ →	• lose if logs used	5
1.3.3.	$3^{\sqrt{y}} = 8$		
	$\sqrt[3]{3^{\sqrt{y}}}$		
	$= (3^{\frac{1}{3}})^{\sqrt{y}}$		
	$= 3^{\frac{1}{3}\sqrt{y}}$ ✓	$\sqrt{}^{-1} \Rightarrow$ exp	
	$= (3^{\sqrt{y}})^{\frac{1}{3}}$		
	$= (8)^{\frac{1}{3}}$		
	$= (2^3)^{\frac{1}{3}}$ ✓		
	$= 2$ ✓ →	ans only 0/3	3
	• argument must be clear + logical		

2.1.	$4 + 1 - 2 - 5 \quad = -10\,875$		
	$\checkmark \quad \checkmark \quad \checkmark$		
	$-3 \quad -3 \quad -3$		
	$d = -3$ ✓ $\qquad a = 4$		
	$S_n = \frac{n}{2}(2a + (n-1)d)$		
	$-10\,875 \overset{\checkmark}{=} \frac{n}{2}(2(4) + (n-1)(-3))$	sub	
	$-21\,750 = n(8 - 3n + 3)$		
	$= n(11 - 3n)$		
	$= 11n - 3n^2$		
	$3n^2 - 11n - 21\,750 = 0$ ✓	std form	
	$(n - 87)(3n + 250) = 0$ ✓	factors	
	$\therefore n = 87$ ✓ or $-\frac{250}{3}$ reject	ans with selection	5
2.2.	$\sum_{k=1}^{5}(x - 3k) = \sum_{k=1}^{8}(x - 3k)$		
	$\sum_{k=1}^{5}(x - 3k) = \sum_{k=1}^{5}(x - 3k) + \sum_{k=6}^{8}(x - 3k)$		
	$0 \overset{\checkmark}{=} \sum_{k=6}^{8}(x - 3k)$	or expansions	
	$0 \overset{\checkmark}{=} x - 18 + x - 21 + x - 24$		
	$63 = 3x$		
	$21 = x$ ✓		3

OR

$$\sum_{k=1}^{5}(x-3k) = \sum_{k=1}^{8}(x-3k)$$

LHS

$= x-3+x-6+x-9+\ldots+x-15$

$a = x-3$

$d = x-6-(x-3)$

$= x-6-x+3$

$= -3$

$n = 5$

$S_5 = \frac{5}{2}\left(2(x-3)+4(-3)\right)$

$= \frac{5}{2}(2x-6-12)$

$= \frac{5}{2}(2x-18)$

$= 5x-45$

RHS

$= x-3+x-6+x-9+\ldots+x-24$

$a = x-3$

$d = -3$

$n = 8$

$S_8 = \frac{8}{2}\left(2(x-3)+7(-3)\right)$

$= 4(2x-6-21)$

$= 4(2x-27)$

$= 8x-108$

But $S_5 = S_8$

$\therefore 5x-45 = 8x-108$

$63 = 3x$

$21 = x$

2.3.

— ; — ; — ; —

-23 -39 -55

-16 -16

$d_2 = 2a$ $d_1 = 3a + b$ $T_{60} = a(60)^2 + b(60) + c$ ✓

$-16 = 2a$ $-23 = 3(-8) + b$ $-28727 = 3600(-8) + 1(60) + c$

$-8 = a$ ✓ $1 = b$ ✓ $13 = c$ ✓

$\therefore T_n = -8n^2 + n + 13$ ✓ 5

3.1.	$S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$ ✓		
	$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$ ✓		
✓	$rS_n - S_n = -a + ar^n$ ✓	LHS and RHS	
	$= ar^n - a$		
	$S_n(r-1) = a(r^n - 1)$ ✓	com factors	
	$S_n = \frac{a(r^n-1)}{r-1} \quad (r \neq 1)$		5
3.2.	$\sum_{k=5}^{22} \frac{3}{4}\left(-\frac{2}{3}\right)^{8-k}$		
	$= -\frac{2}{9} + \frac{1}{3} - \frac{1}{2} \quad \dots \left(\frac{3}{4}\right)\left(-\frac{2}{3}\right)^{-14}$		
	$S_n = \frac{a(r^n-1)}{r-1}$		
	$a = -2/9 \quad r = -3/2$ ✓	a r	
	$n = 22 - 5 + 1$		
	$= 18$ ✓	n	
	$\therefore S_{18} = \frac{-\frac{2}{9}\left(\left(-\frac{3}{2}\right)^{18} - 1\right)}{-\frac{3}{2} - 1}$ ✓	form + sub	
	$= 131,28$ ✓	. ans only 1/5	5

3.3.	$(5x+2)+(2-4x)+(x+7)+\dots$		
3.3.1.	$\frac{2-4x}{5x+2} = \frac{x+7}{2-4x}$ ✓	ratios =	
	LCD $= (5x+2)(2-4x)$		
	$(\therefore x \neq -2/5 \text{ or } \frac{1}{2})$		
	x thru		
	$(2-4x)(2-4x) = (x+7)(5x+2)$		
	$4 - 16x + 16x^2 = 5x^2 + 37x + 14$		
	$11x^2 - 53x - 10 = 0$ ✓	std form	
	$(x-5)(11x+2) = 0$ ✓	factors	
	$\therefore x = 5 \text{ or } -2/11$ ✓	both	4
3.3.2.	$x=5: (5(5)+2)+(2-4(5))+(5+7)+\dots$		
	$\therefore 27 - 18 + 12 \dots r = -2/3$ ✓		
	accept as $-1 < r < 1$		
	$x = -\frac{2}{11}: (5(-\frac{2}{11})+2)+(2-4(-\frac{2}{11}))+(-\frac{2}{11}+7)+\dots$		
	$\therefore \frac{12}{11} + \frac{30}{11} + \frac{75}{11} + \dots r = \frac{5}{2}$		
	$r > 1 \therefore$ reject ✓ $x = -\frac{2}{11}$	rejection	
	$\therefore S_\infty = \frac{27}{1-(-2/3)}$ ✓	form + sub	
	$= \frac{81}{5}$ ✓ 16,2		4

3.4	$T_1 + T_2 + T_3 = 17$		
	$a + ar + ar^2 = 17$ ✓		
	$T_6 + T_7 + T_8 = 544$		
	$ar^5 + ar^6 + ar^7 = 544$ ✓		
	$r^5(a + ar + ar^2) = 544$ ✓	com fact	
	$r^5 \cdot 17 = 544$		
	$r^5 = 32$		
	$r = \sqrt[5]{32}$		
	$r = 2$ ✓ →		4
4.	$S_1 = \frac{1}{4}$		
	$S_2 = \frac{1}{4} + \frac{11}{20} = \frac{4}{5}$		
	$S_3 = \frac{1}{4} + \cdots + \frac{7}{10} = \frac{3}{2} \times \frac{3}{3} = \frac{9}{6}$		
	$S_4 = \frac{1}{4} + \cdots + \frac{11}{14} = \frac{16}{7}$		
	$S_5 = \frac{1}{4} + \cdots + \frac{47}{56} = \frac{25}{8}$		
	$\therefore S_n = \frac{n^2}{n+3}$ ✓ ✓ →	num dem	2

5.1. $f: y = -\frac{3}{x+4}$

5.1.1. $x \in \mathbb{R}; x \neq -4$ ✓ → 1

5.1.2. ha: $y = 0$ ✓ → eq ✓

va: $x = -4$ ✓ → eqn ✓ → 2

5.1.3. y int: $y = -\frac{3}{4}$

x int: $0 = -\frac{3}{x+4}$

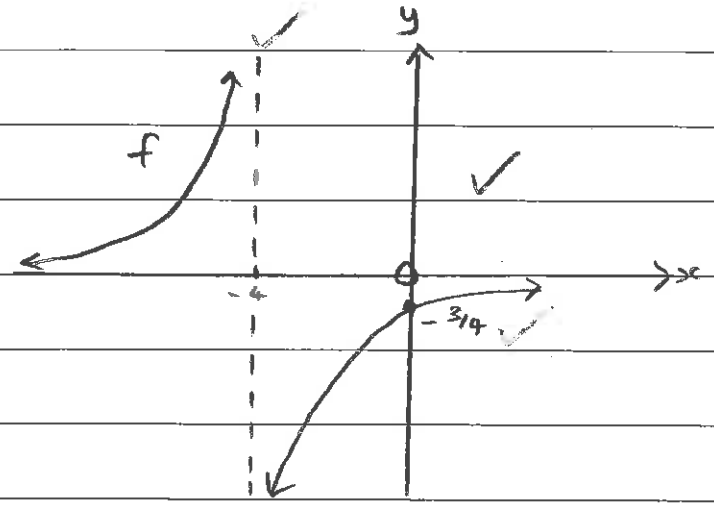
$LCD = (x+4) \quad (\because x \neq -4)$

$x+4 \neq 0$

$0 \neq -3$

no soln $\therefore$ no x int

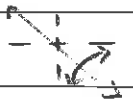
shape: $a = -3$ $\quad -\ | \ +$



va

shape + ha

y int → 3

5.1.4	g: $y = \frac{3}{x+4}$ ✓		1
5.1.5.	$y = -(x+4)$		
	$\therefore y = -x - 4$ ✓ ✓	✓ $-x-4$ ½	2
5.2.	$y = -x + 5$		
	$x = -7$ $\quad y = 9$		
	$y = -(-7) + 5$ | $9 = -x + 5$		
	$= 12$ | $x = -4$		
	✓ ✓ $\therefore (-4; 12)$		2
5.3.	$y = \frac{3-4x}{x+5}$		
	$x+5 \overline{)-4x+3}$ ✓ $\quad -4$	method	
	⊕ ⊖ $-4x-20$		
	$+23$		
	$\therefore y = -4 + \frac{23}{x+5}$ ✓	accept	
	$= \frac{23}{x-(-5)} + (-4)$		2

5.4	VA: $x = -2$ $\therefore x + 2 = 0$		
	$\therefore y = \frac{3x - m}{x + 2}$		
	Sub $A(-\frac{2}{3}; 0)$		
	$0 = \frac{3(-\frac{2}{3}) - m}{-\frac{2}{3} + 2}$		
	$0 = \frac{-2 - m}{4/3}$		
	$\times \frac{4}{3}: 0 = -2 - m$		
	$m = -2$ ✓		
	$x + 2 = x + k$		
	$\therefore 2 = k$ ✓		2

6.1.1.	(a) $A(-2;-18)$ ✓	both coords	1
	(b) x int: $0 = -2x+2$		
	$2x = 2$		
	$x = 1$		
	$\therefore B(1;0)$ ✓	1 coord	1
	(c) $\frac{x_c + 1}{2} = -2$		
	$\times 2$: $x_c + 1 = -4$		
	$x_c = -5$		
	$\therefore C(-5;0)$ ✓	1 coord	1
6.1.2.	$y = a(x+2)^2 - 18$ ✓		
	sub $B(1;0)$		
	$0 = a(1+2)^2 - 18$ ✓	sub	
	$18 = 9a$		
	$2 = a$ ✓	a	
	$\therefore y = 2(x+2)^2 - 18$		
	$= 2(x^2+4x+4) - 18$	$\times$ out brackets	
	$= 2x^2 + 8x + 8 - 18$		
	$= 2x^2 + 8x - 10$		4

	(OR)		
	$y = a(x+5)(x-1)$ ✓		
	sub $A(-2;-18)$		
	$-18 = a(-2+5)(-2-1)$ ✓		
	$= a(3)(-3)$		
	$= -9a$ ✓		
	$2 = a$		
	$\therefore y = 2(x+5)(x-1)$		
	$= 2(y^2 + 4x - 5)$ ✓		
	$= 2x^2 + 8x - 10$		
6.1.3.	(a) $y = 2x^2 + 8x - 10$ $y = -2x + 2$		
	$2x^2 + 8x - 10 = -2x + 2$ ✓	equate	
	$2x^2 + 10x - 12 = 0$		
	$\div 2$: $x^2 + 5x - 6 = 0$ ✓	std form	
	$(x-1)(x+6) = 0$ ✓	factors	
	$\therefore x =$ 1 or -6 (reject)		
	$\therefore y = -2(-6) + 2$		
	$= 14$		
	$\therefore D(-6;14)$	x y	5

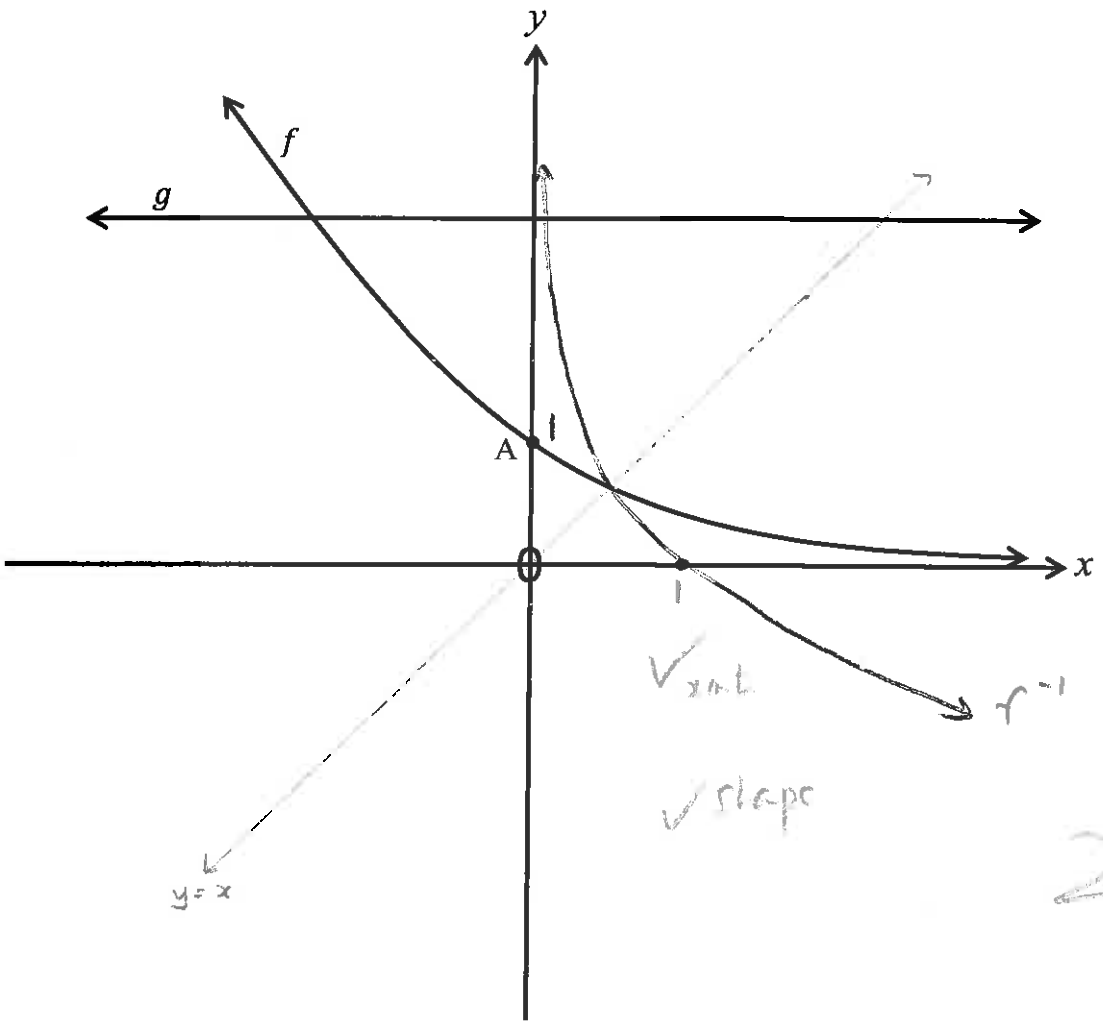
6.13. (b)	$PQ = y_P - y_Q$		
	$= -2x+2-(2x^2+8x-10)$ ✓	g - f method	
	$= -2x+2-2x^2-8x+10$	• $f = \frac{49}{2}$ • $g = 49$ 0/5	
	$\frac{49}{2} = -2x^2-10x+12$	• assume @ PQ_{max} 3/5	
	$\times 2$: $49 = -4x^2-20x+24$		
	$4x^2+20x+25=0$ ✓	Std form	
	$(2x+5)(2x+5)=0$ ✓	factors	
	$x = -5/2$		
	$\therefore y_P = -2(-5/2)+2$		
	$= 7$		
	✓ ✓		
	$\therefore P(-5/2; 7)$	x y	5
6.14 (a)	$x \cdot f(x) > 0$		
	x y_f +		
	✓ ✓		
	$\therefore x \in (-5;0)$ or $(1;\infty)$		2
(b)	$\frac{f(x)}{g(x)} \leq 0$		
	$\frac{y_f}{y_g} \leq 0$		
	✓ ✓		
	$\therefore x \in [-5;1)$ or $(1;\infty)$		2

	(OR)		
	$x \in [-5; \infty)$; $x \neq 1$		
6.2.	$y = ax^2 + bx + c$		
	$a = -$ $b = +$ $c = -$		
	$\curvearrowright$ $x = p$ y-int $-$		
	$= \frac{-(+)}{2(-)}$		
	$= +$		
	y		
	✓	AOS +	
	✓ ✓ x	tp on x-axis	
	✓	y-int < 0	
	✓	shape	
			4
	$b^2 - 4ac = 0$		
	$\Delta = 0$		
	Equal roots ∴ turning point on x-axis		

Name and Surname : Solno .....

QUESTION 7 - ANSWER SHEET

7.1 In the diagram below, $f(x) = \left(\frac{1}{3}\right)^x$ and $g(x) = 3$.



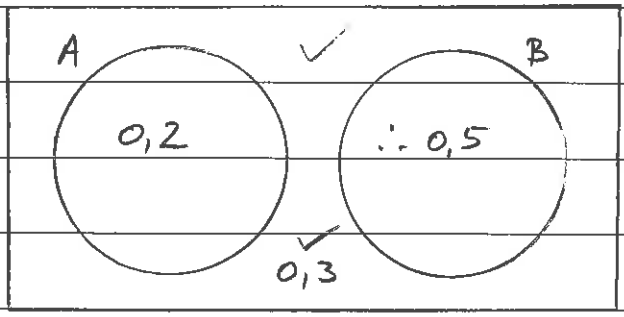
2

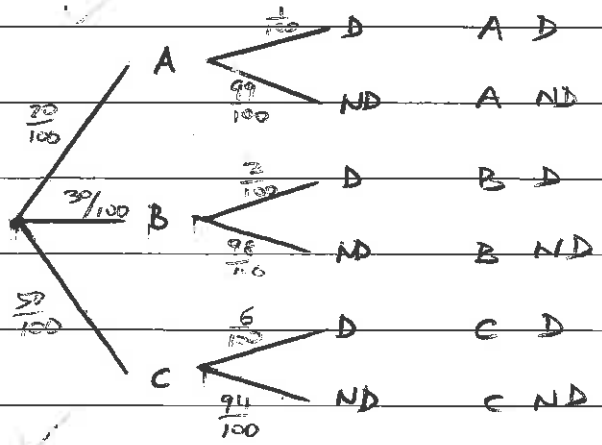
7.1.1	A(0;1) ✓ ✓ $(\frac{1}{3})^0 = 1$	1
7.1.2.	See graph	

7.1.3.	$\log_{\frac{1}{3}} x = 3$	
	$\left(\frac{1}{3}\right)^3 = x$	
	$\frac{1}{27} = x$ ✓	1
7.1.4.	$\log_{\frac{1}{3}} x \geq 3$	$f: y = \left(\frac{1}{3}\right)^x$
	$f^{-1} \geq 3$	$f^{-1}: x = \left(\frac{1}{3}\right)^y$
	$\therefore x \in (0; \frac{1}{27}]$ ✓ ✓	$y = \log_{\frac{1}{3}} x$ 2
7.2.	$h: y = -\sqrt{-3x}$	
	$h^{-1}: x = -\sqrt{-3y}$ ✓ $x \leftrightarrow y$	
	$(x)^2 = \left(-\sqrt{-3y}\right)^2$	
	$x^2 = -3y$	
	$-\frac{1}{3}x^2 = y$	
	$\therefore y = -\frac{1}{3}x^2 \quad (x \leq 0)$ ✓ ✓	3

8.1.	$A = P(1-i)^n$		
	$x = 2x\left(1-\frac{12}{100}\right)^n$ ✓	form + sub	
	$\frac{1}{2} = \left(\frac{22}{25}\right)^n$		
	$n = \frac{\log\left(\frac{1}{2}\right)}{\log(22/25)}$ ✓	use of logs	
	$= 5,42$ years ✓		3
	(6 full years)		
8.2.	$1 + i_{ea} = \left(1 + \frac{i_{nom}}{1c}\right)^k$		
	$k = 12$ ✓	k	
	$1 + \frac{15}{100} = \left(1 + \frac{i_{nom}}{12}\right)^{12}$ ✓	form + sub	
	$\sqrt[12]{\frac{23}{20}}$ ✓ $= 1 + \frac{i_{nom}}{12}$	$\sqrt[12]{}$	
	$1,011\ldots =$		
	$0,14\ldots = i_{nom}$		
	$I_{nom} = 14,06\%$ pa (comp monthly)		4
8.3	$A = P(1+i)^n$		
	$1.1.2019 \rightarrow \overset{31.12.2030}{1.1.2031}$ $\therefore 2031-2019 = 12$	$i = \frac{7}{1200}$ ✓ $n = 12\times12 = 144$ ✓	
	$A = 1500\left(1+\frac{7}{1200}\right)^{144}$ ✓	form + sub	
	$= R\ 3466,08$ ✓		4

9	$f(x) = 30x^3 - 49x^2 + 9x + 4$		
9.1.	$2x - 1 = 0 \quad \therefore \quad x = \frac{1}{2}$		
	$R = f(\frac{1}{2})$		
	$= 30(\frac{1}{2})^3 - 49(\frac{1}{2})^2 + 9(\frac{1}{2}) + 4$ ✓	Sub $x = \frac{1}{2}$	
	$= 0$ ✓		2
	$\therefore$ $2x - 1$ is a factor ⟶		
9.2.	$30x^3 - 49x^2 + 9x + 4$		
	$= (2x - 1)(15x^2 - 17x - 4)$ ✓✓	$15x^2 - 4$ $-17x$	
	$-15x^2$ $-34x^2$		
	$= (2x - 1)(3x - 4)(5x + 1)$ ✓ ⟶		3

10.1.	$P(A \cup B) + P((A \cup B)') = 1$		
	$P(A \cup B) + 0,3 = 1$		
	$P(A \cup B) = 0,7$ ✓		
	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$		
	$0,7 = 0,2 + P(B) - 0$ ✓		
	$0,5 = P(B)$ ✓		3
	(OR)		
	$S = 1$		
		no ∩ $P((A \cup B)')$ correctly represented	
	$\therefore P(B) = 0,5$ ✓		
10.2.1.	$x + 0,3 + 2x + 0,1 = 1$ ✓	$S = 1$	
	$3x = 0,6$		
	$x = 0,2$		1

10.2.2.	$P(A) = 0,2 + 0,3 = 0,5$ ✓	P(A) NOT P(A only)	
	$P(B) = 0,3 + 2(0,2) = 0,7$ ✓	P(B) NOT P(B only)	
	$\therefore P(A) \times P(B) = 0,5 \times 0,7$ ✓	product	
	$= 0,35$		
	$P(A \cap B) = 0,3$		
	$\therefore P(A \cap B) \neq P(A) \times P(B)$ ✓	must be stated	
	$\therefore$ A and B are NOT ✓	conclusion	
	independent		5
10.3.1.		Use one + probs	
		Use two + probs	
		Outcomes	
	✓	Shape	4
10.3.2	(a) $P(B\ ND)$		
	$= \frac{30}{100} \times \frac{98}{100}$ ✓	product	
	$= \frac{147}{500}$ ✓ 0.29		2

10.3.2.	(b) $P(D)$		
	$= \frac{20}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{2}{100} + \frac{50}{100} \times \frac{6}{100}$	✓ all products ✓ addition	
	$= \frac{1}{500} + \frac{3}{500} + \frac{3}{100}$		
	$= \frac{19}{500}$ ✓ 0,04		3